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Position and Quantum Theory

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It is customarily maintained that the concept of unmeasured (i.e., unobserved) position is without meaning in quantum theory. However, the vast literature on the subject, as shown here through the typical example of the double-slit experiment, really does not provide an iron-clad proof that this is indeed so. The question of the possible meaning of unobserved position is thus open. The problem is then resolved by showing quite rigorously that, after all, unobserved position is meaningless in quantum theory. The criterion for unobserved position to be meaningful in any theory (probabilistic or not) is taken to be that the probability density for position must satisfy the Einstein-Chapman-Kolmogorov equation with a positive-semidefinite kernel which is also properly normalized.

1. INTRODUCTION

One of the most unusual and conceptually difficult statements (accepted by many) emerging from the background experiments of quantum theory as well as the formalism of quantum mechanics itself is that the notion of position of an object only has meaning at the moment of measurement; i.e., unmeasured (that is, unobserved) position is a meaningless concept. For example, we are told that the pattern of particles arriving at a screen after coming undisturbed through a double-slit arrangement is incompatible with those particles having meaningful positions between source and screen. The argument here is so basic that it has even been carried out without the formalism of quantum mechanics. Of course, it can also be completed with the quantum formalism as well.

Such astonishing statements must certainly be investigated in the greatest detail before being accepted and, of course, these statements have been the subject of a great deal of discussion in the literature since the earliest days of quantum theory. However, as many know who have read this material, the argumentation found there is often not very compelling. This is, in part, because frequently no careful distinction is made between the consequences of experiments and the consequences of the quantum

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mechanical formalism. It is also due to the fact that the consequences drawn from the formalism itself are sometimes not sharp and are subject to diverse interpretations. So, for many, the problem remains—does unobserved position have meaning?

In this work we shall ultimately not be concerned with consequences from experiment itself, but shall finally focus entirely on the consequences of the quantum formalism.

In the following discussion I shall first briefly try to convince the reader that there really is an unsolved problem here by choosing a well-worn example from traditional considerations for analysis; I shall consider the quantum mechanical analysis of the double-slit experiment. The goal of this development will certainly not be a full coverage of the pros and cons of the position-meaning problem, but merely to present sufficient considerations that the reader may question some of the traditional arguments.

Finally, in the second part of the paper a rigorous, rather novel approach to the problem will be used to demonstrate that, after all, the formalism of quantum mechanics decidedly does imply that unobserved position is meaningless. This will be shown by proving that essentially any probability density $\rho = \psi\psi^*$, where ψ is governed by the Schrödinger equation, cannot satisfy the Einstein–Chapman–Kolmogorov (ECK) equation with an everywhere nonnegative transition probability which is also properly normalized. These latter requirements, as will be discussed, are taken as the criterion for unobserved position to be meaningful in any theory.

2. DOUBLE-SLIT EXPERIMENT

Here, I reconsider the well-known double-slit experiment, with the attendant quantum formalism. Consider then a beam of independent particles incident on the customary double-slit arrangement, the particles finally forming the characteristic pattern on the screen. Assume that, between the particle source and the screen, the particles are not disturbed in any way. The quantum formalism may then be used to construct an argument which *apparently* proves that the shape of the observed particle pattern on the screen contradicts the hypothesis that the unobserved particles, between source and screen, may be considered as having definite though unknown positions at each moment. This traditional argument goes as follows.[2] Assuming for simplicity that the slits are sufficiently narrow, we can write for the wave function at the screen, via the Feynman path-integral approach,

$$\psi(\mathbf{x}, t) = K_1\psi(\mathbf{x}_1, t_1) + K_2\psi(\mathbf{x}_2, t_1) \tag{2.1}$$

where $\mathbf{x}$ denotes any location on the screen; $\mathbf{x}_1$ and $\mathbf{x}_2$ denote the slit

[2]This traditional argument may be found, for example, in Bohm (1951); see also D'Espagnat (1976) and Feynman and Hibbs (1965).

locations; K_1 and K_2 are Feynman propagators between the slits and screen, and are defined as transition amplitudes from slit to screen for *observed* particles; and $\psi(\mathbf{x}_1, t_1)$ and $\psi(\mathbf{x}_2, t_1)$ denoting the incoming wave function at the individual slits 1 and 2, respectively, at time $t_1 < t$. We then get the following observationally correct expression for probability densities at the screen:

$$\rho(\mathbf{x}, t) = |K_1|^2 \rho(\mathbf{x}_1, t_1) + |K_2|^2 \rho(\mathbf{x}_2, t_1) + 2\,\mathrm{Re}\{K_1 K_2^* \psi(\mathbf{x}_1, t_1)\psi^*(\mathbf{x}_2, t_1)\}$$

where Re signifies the real part. Note that, if just slit 1 or just slit 2 were open, then we would have just the first or just the second term present on the right-hand side here, respectively.

The argument is then made that (Bohm, 1951, p. 122), "if the experiment involved a probability distribution of classical particles" we would have, instead,

$$\rho(\mathbf{x}, t) = |K_1|^2 \rho(\mathbf{x}, t_1) + |K_2|^2 \rho(\mathbf{x}_2, t_1) \tag{2.3}$$

holding as the pattern on the screen. The argument is concluded by noting that equation (2.3) differs from the correct relation (2.2) through lack of "interference terms." Thus, the conclusion is forced that unobserved position must be meaningless.

Now, looking at this argument more closely, we see that it is weak in assuming that equation (2.3) correctly describes the particle pattern to be expected on the screen in case unobserved position is meaningful. This is so since equation (2.3) can only be expected to be true if the (unobserved) particles each go through one slit or the other. But it is quite conceivable that the particles may have unobserved though definite position at each moment and *not* go through one slit or the other. This *could* happen, for instance, if the unobserved position were not a continuous function of time. We know that very precise successive position measurements on objects certainly do not support any *observed* behavior that is truly continuous in time. Therefore, even in our conceptualizing about unobserved (though definite) particle position we have no justification in assuming it to be continuous in time. Thus, from this new viewpoint, we might expect each particle to interact, in some sense, with *both* slits. Or, expressed differently, the slits would not behave independently in their effect on each particle. We would then expect a relation of the *form* of equation (2.3) to hold, but with the $|K_i|^2$ replaced by other quantities which would each reflect the synergism of the two slits. These new quantities would then represent the transition probabilities from slit to screen of the *unobserved* particles.[3]

[3]In connection with equation (2.3), also note that we have no right to assume that the $|K_i|^2$ (which are defined in terms of certain *observations*) in a situation where observations are actually *not* made (i.e., at the slits here) are equivalent to transition probabilities that describe particles with *unobserved* but meaningful positions.

Therefore, equation (2.3) may not correctly describe particles with unobserved though definite positions. And if this equation is suspect, then, of course, so is the conclusion of the traditional argument.

Although the above argument was only fabricated to point out a possible weakness in the traditional analysis, it is hoped that the reader at this point will be convinced that the position-meaning has certainly not been resolved by the customary quantum arguments.

Finally, as a preparatory comment, I note that $\rho(\mathbf{x}, t)$ as given in equation (2.2) can be expressed differently, in a form, related to the above fabrication, which will be basic for later considerations. That is, we can rewrite equation (2.2) as

$$\rho(\mathbf{x}, t) = \sum_j \left\{ \frac{1}{2} \left[\sum_i \left(K_i K_j^* \frac{\psi_i}{\psi_j} + K_i^* K_j \frac{\psi_i^*}{\psi_j^*} \right) \right] \right\} \psi_j \psi_j^* \tag{2.4}$$

where the summation is over the two slits, and where $\psi_i \equiv \psi(\mathbf{x}_i, t_1)$.

In this form, $\rho(\mathbf{x}, t)$ *might* be considered as a sum, over mutually exclusive alternatives, of *unobserved* transitions (where the real quantity in parentheses plays the role of a "transition probability") from the two slits, and where $\psi_j \psi_j^*$ is assumed, of course, to also be equal to the probability density of unobserved particles. Here, the "transition probabilities" which display the interdependence of the slits, if all nonnegative, could be interpreted as merely reflecting the contributions that particles (with definite though unobserved positions) near one slit ultimately make to the pattern on the screen. In this case the "transition probabilities" would be the quantities that correctly replace the $|K_i|^2$ in equation (2.3), And, if the "transition probabilities" are ever negative, this would betray the fact that unobserved position is meaningless.

The rest of this work will, in fact, be devoted—in a more general setting—to the question of whether or not ρ can be represented as in equation (2.4) with "transition probabilties" that are always nonnegative and properly normalized. I shall show that, for essentially any $\rho = \psi\psi^*$, where ψ is governed by the Schrödinger equation, this is not possible.

3. CRITERION OF MEANING

In this section I formulate the criterion for unobserved position to be meaningful in any theory. For simplicity, I make the considerations one-dimensional.

If unobserved position is to be meaningful, then one should be able to speak of the probability density that unobserved particles should have some position or other. I assume that position measurements (i.e., observations) do not *locally* perturb the unobserved positions of such particles (their subsequent behavior may certainly be affected, however) so that one can identify the probability density for unobserved particles with that for observed particles at any location. Thus, when I apply these considerations to quantum mechanics later, I shall consider $\rho = \psi\psi^*$ to refer to the probability density of both observed or unobserved particles. (I am using the term "observed" in the above several lines to refer to results that would occur *if* one decided to make an observation; this, however, does not mean that such observations are actually made. In fact, when one speaks of the time evolution of ρ in quantum theory, it is understood that prior to any time of interest, such observations, after the initial preparation of the state, are not actually made.)

Now, if unobserved position is meaningful, then it must be that the probability density at any location at some time can be considered as composed of contributions coming from all possible locations at any given earlier time. That is, I take as the criterion for unobserved position to have meaning in any theory that the probability density satisfies the Einstein-Chapman-Kolmogorov (ECK) equation, which is expressed in one dimension as

$$\rho(x, t) = \int Q(xt; zt_1)\rho(z, t_1)\, dz \tag{3.1}$$

where $t_1 \leq t$, and Q is the transition probability that unobserved particles go from z to x in the time interval $t - t_1$. Further, in order to be a *true* (i.e., physical) transition probability, Q must also satisfy the conditions

$$\text{(i)} \quad Q(xt; zt_1) \geq 0 \qquad \text{for all} \quad x, z, t \geq t_1 \tag{3.2}$$

$$\text{(ii)} \quad \int Q(xt; zt_1)\, dx = 1 \qquad \text{for all } z, t \geq t_1 \tag{3.3}$$

and one says that the latter condition describes a *properly normalized* kernel.[4]

In the following sections, I shall show that essentially any $\rho = \psi\psi^*$, where ψ is governed by the Schrödinger equation, cannot satisfy the ECK

[4]Feynman (1987) recently considered the interesting formal possibility that probabilities, in certain cases, may be negative. He does not ascribe any particular physical significance to such probabilities and assumes that they are only permissible "in the sense that the assumed conditions of preparation or verification are experimentally unattainable." That is, situations described by negative probabilities cannot be prepared or else cannot be verified experimentally, but they may be useful in describing imagined intermediary states. The transition probability Q [see equation (3.1)] that we consider in this paper would qualify as such a probability. However, aside from this there is no significant overlap between Feynman's considerations and the present ones, since Feynman is concerned only with the possible existence of negative probabilities and I am concerned with formulating a criterion of meaning for unobserved position, which, incidentally, involves quantities (like Q) which may be interpreted as one of Feynman's negative probabilities.

equation with Q satisfying the above two conditions. In this way, then, I demonstrate that unobserved position is meaningless in quantum theory.

4. POSITION AND QUANTUM THEORY

In this section I shall apply the criterion for meaningfulness of position to quantum theory, where I remind the reader that I am making one-dimensional considerations. The problem, then, is this: given the quantum probability density $\rho(x, t)$, for all x and t, where $\rho = \psi\psi^*$ with ψ being governed by the Schrödinger equation, does there then exist a Q such that

$$\rho(x, t) = \int Q(xt; zt_1)\rho(z, t_1)\, dz \tag{4.1}$$

for all x, $t \geq t_1$, where $Q \geq 0$ always, and where also Q is properly normalized?

To solve this problem one must first make general considerations concerning the construction of the kernel Q so that it satisfies the ECK equation. There are two general methods for constructing such kernels. With the first method I shall use what I shall refer to as identity transformations, and with the second method, I shall consider what I shall call substantive transformations.

4.1. First Method

Using consequences of the Schrödinger equation, one can easily construct a candidate for Q as follows: From the path-integral formulation of quantum mechanics we have, for any ψ,

$$\psi(x, t) = \int K(xt; zt_1)\psi(z, t_1)\, dz \tag{4.2}$$

for all x, $t \geq t_1$, where K here is the appropriate propagator. This gives for the probability density

$$\rho(x, t) = \int \left\{ \int K(xt; zt_1)K^*(xt; yt_1) \frac{\psi^*(y, t_1)}{\psi^*(z, t_1)}\, dy \right\} \rho(z, t_1)\, dz \tag{4.3}$$

where we see that the expression in parentheses on the right-hand side plays the role of a (complex) "transition probability." We can, however, rewrite this so that the kernel is real, and we have

$$\begin{aligned} \rho(x, t) &= \int \frac{1}{2} \left\{ \int \left[K(xt; zt_1)K^*(xt; yt_1) \frac{\psi^*(y)}{\psi^*(z)} + \text{c.c.} \right] dy \right\} \rho(z, t_1)\, dz \\ &\equiv \int P\rho\, dz \end{aligned} \tag{4.4}$$

for all x, $t \geq t_1$, where I have suppressed the t_1 dependence in ψ for simplicity, and where I have denoted the above kernel by the specific letter P because of its importance for later discussion. I also note here that the transformation from the right-hand side of equation (4.3) to the right-hand side of equation (4.4) holds for *any* functions K and ψ.

Thus, quantum theory gives us a candidate for the ECK equation. (Whether or not P also satisfies the requisite additional two conditions is another matter, which I shall consider in detail later.)

A question that must be immediately addressed is, is the above real kernel unique? That is, *for a given* K, are there other ways of arranging or rewriting or introducing terms so that we might have, *for the same* $\rho(z, t_1)$ and $\rho(x, t)$,

$$\rho(x, t) = \int Q(xt; zt_1)\rho(z, t_1)\, dz \tag{4.5}$$

for all x, $t \geq t_1$, where Q is some other real combination of the K's and ψ's. An example of this kind of occurrence was in the transition from the (complex) kernel in equation (4.3) to the (real) kernel in equation (4.4). For the kind of changes we are considering here, the kernels may be said to be related by an "identity transformation." That is, $\int P\rho\, dz$ and $\int Q\rho\, dz$ are the same for *any* functions K and ψ, even though $P \not\equiv Q$. Of course, only for *certain* K (determined by the Schrödinger equation) is $\int P\rho\, dz = \rho(x, t) = \int Q\rho\, dz$.

Now, suppose that besides P there is another kernel Q satisfying the ECK equation, where both kernels are constructed from the same K and ψ, i.e., they are related by an identity transformation. Then we must have

$$\int (P - Q)\rho\, dz = 0 \tag{4.6}$$

as an *identity*, i.e., for *any* functions K and ψ. But, as I shall show, this implies that Q must be of the same degree in the K's, K^*'s, ψ's, and ψ^*'s as is P. For example, suppose that

$$Q = \iint \left\{ K(x, t; zt_1)K^*(xt; yt_1)K(xt; \lambda t_1)\frac{\psi^*(y)}{\psi^*(z)}\psi(\lambda) + \text{c.c.} \right\} dy\, d\lambda \tag{4.7}$$

and put this expression, together with the expression for P, into equation (4.6). The resulting relation must hold for *all* K and ψ. Now, I shall take particular functional derivatives of this relation about the arbitrary K, but do not vary the ψ's. If one writes $K = A\, e^{iB}$, then one takes $\delta K = e^{iB}\delta A = K\delta A/A$ and $\delta K^* = K^*\delta A/A$. I then apply $\delta/\delta K(xt; \xi t_1)$ to both sides of the above relation. The details are too long to present here, so I comment on them in general terms. I then next apply $\delta/\delta K(xt; \xi' t_1)$ to the resulting equation that one has after the first functional derivation. At this point one has a sum of terms, some consisting of products of the K, K^*, ψ, and ψ^*, each evaluated at ξ or ξ', and a sum of terms each consisting of a single integral. When one finally applies $\delta/\delta K(xt;\ \xi'' t_1)$ to this relation only the terms consisting of integrals (which all come from Q) contribute. We then end up with a long sum of similar terms, each being a product of three K's (including one or two K^*'s) and three ψ's (including one or two ψ^*'s). Now choose $\xi' = \xi'' \equiv \xi$ to be arbitrary, and also choose ψ to be real (this is alright, as K is independent of ψ). Further choosing $\psi(\xi)$ to never vanish, we can then divide out by the products of the ψ's, and obtain,

$$K(xt;\ \xi t_1)K(xt;\ \xi t_1)K^*(xt;\ \xi t_1) + K^*(xt;\ \xi t_1)K^*(xt;\ \xi t_1)K(xt;\ \xi t_1) = 0 \quad (4.8)$$

Consequently, we must have

$$K(xt;\ \xi t_1) + K^*(xt;\ \xi t_1) = 0 \quad (4.9)$$

for all ξ. And this relation implies that the real part of K must vanish for all ξ.

Now, let us reconsider the whole development above, but with a different variation. This time take $\delta K = i\delta BK$ and $\delta K^* = -i\delta B \cdot K^*$. These variations have the same general form as the ones used before, except for the important difference that δK^* involves a different sign than δK. If one then repeats the previous procedure with this variation, one gets the same terms except that they have different coefficients than before. Whereas before all terms occurred multiplied by -1, now they all occur with plus signs and in addition some are multiplied by i. We now get the relation

$$iK(xt;\ \xi t_1) + K^*(xt;\ \xi t_1) = 0 \quad (4.10)$$

for all ξ. And this relation, together with the already established result that $\mathrm{Re}(K) = 0$, implies that $K = 0$ for all ξ. Thus, equation (4.6) is trivial, and such a Q does not exist. Only if P and Q depend on the K's to the *same degree* (i.e., the K's and K^*'s to the same powers) will this not happen. In that case we would obtain, instead, relations between the coefficients of the various products of the K's involved in P and Q. Finally, everything just concluded about the K's here also holds, by exactly the same reasoning for the ψ's. Therefore, the assertion is proven.

We now see that if there is another (real) kernel Q which is related by an identity transformation to P, then Q must be of the same degree in K, K^* and ψ, ψ^* as P is. So, Q must have the same structure as P, except possibly with different coefficients. But it is easy to see then that no other such properly normalized kernel exists besides P itself.

4.2. Second Method

In this section I consider the other way that one might construct different kernels; this is by actually modifying K so that $\psi(x, t)$ remains the same. For example, suppose we have

$$\psi(x, t) = \int K(xt; zt_1)\psi(z, t_1)\, dz \tag{4.11}$$

for some given K, for all $x, t \geq t_1$. We can easily construct another kernel $\bar{K}$ here which satisfies the same equation for all $x, t \geq t_1$, by defining $\bar{K} \equiv K(xt; zt_1) + G(xt; zt_1)$, where G satisfies the relation

$$\int G(xt; zt_1)\psi(z, t_1)\, dz = 0 \tag{4.12}$$

for all $x, t \geq t_1$. Then, we obviously have

$$\psi(x, t) = \int \bar{K}(xt; zt_1)\psi(z, t_1)\, dz \tag{4.13}$$

for all $x, t \geq t_1$.

Now let us see what further consequences follow for $\rho(x, t)$ from this approach. From equation (4.13) we have

$$\rho(x, t) = \frac{1}{2}\int\left\{\int\left[\bar{K}(xt; zt_1)\bar{K}^*(xt; yt_1)\frac{\psi^*(y)}{\psi^*(z)} + \text{c.c.}\right] dy\right\}\rho(z, t_1)\, dz \tag{4.14}$$

Now substituting the relation $\bar{K} = K + G$ here and using equation (4.12) yields the relation

$$\rho(x, t) = \int [P(xt; zt_1) + \Delta(xt; zt_1)]\rho(z)\, dz \equiv \int Q^{(s)}(xt; zt_1)\rho(z)\, dz$$

where

$$\Delta(xt; zt_1) \equiv \mathrm{Re}\left\{\int G(xt; zt_1)K^*(xt; yt_1)\frac{\psi^*(y)}{\psi^*(z)}\, dy\right\} \tag{4.16}$$

for a G satisfying equation (4.12), and the superscript s on Q above distinguishes this "substantive" construction from that considered in the previous section.

5. Q IS NOT PHYSICAL

In the preceding sections I have discussed the two ways that one can generate kernels in quantum theory so that the kernel satisfies the ECK equation (3.1). In this section I shall show that, in fact, neither way can produce a kernel which also satisfies the requisite conditions (3.2) and (3.3) which would make the kernel a *physical* transition probability. Thus, I shall conclude that position is meaningless in quantum theory.

I first address the construction of the kernel $Q^{(s)}$. Throughout the rest of the discussion I restrict consideration to the case where there is no external field; $V(x) \equiv 0$. This case will be sufficient for present purposes, since the result is in the negative.

Then, paraphrasing conditions (3.2) and (3.3), to make $Q^{(s)}$ physical, we must find a G such that these conditions hold:

$$\text{(i)} \quad P_0 + \Delta \geq 0 \qquad \text{for all} \quad x, z, t \geq t_1 \tag{4.17}$$

$$\text{(ii)} \quad \int \Delta(xt; zt_1)\, dx = 0 \qquad \text{for all} \quad z, t \geq t_1 \tag{4.18}$$

since, as is easily shown,[5] $\int P_0(xt; zt_1)\, dx = 1$, where the zero subscript is to remind us of zero external field, and,

$$\text{(iii)} \quad \int G(xt; zt_1)\psi(z, t_1)\, dz = 0 \tag{4.19}$$

where Δ is defined in terms of G according to equation (4.16).

I also assume that $\bar{K}_0 \equiv K_0 + G$, and therefore G is continuous in x and z as is K_0 itself. This is a very reasonable assumption, as there should be no reason for discontinuities in empty space.

From equation (4.16) and condition (i) above, we have

$$\begin{aligned} &\mathrm{Re}\left\{\int K_0^*(xt; yt_1) r(y)\, e^{-i/\hbar)\varphi(y)} dy\right\} \\ &\quad \times \mathrm{Re}\{G(xt; zt_1) + K_0(xt; zt_1\} \\ &\quad \geq \mathrm{Im}\left\{\int K_0^*\,(xt; yt_1) r(y)\, e^{-(i/\hbar)\varphi(y)}\, dy\right\} \\ &\qquad \times \mathrm{Im}\{G(xt; zt_1) + K_0(xt; zt_1)\} \end{aligned} \tag{4.20}$$

[5]Using the expression for the free kernel K_0 [as given in equation (4.21] in P_0, we obtain

$$\begin{aligned} \int P_0(xt; zt_1)\, dx &= \frac{\alpha}{\pi}\,\mathrm{Re}\left\{\int \frac{\psi(y)}{\psi(z)}\, e^{i\alpha(y^2 - z^2)}\left[\int e^{2i\alpha x(z-y)} dx\right] dy\right\} \\ &= \mathrm{Re}\left\{\int \frac{\psi(y)}{\psi(z)}\, e^{i\alpha(y^2-z^2)})\, \delta(z-y)\, dy\right\} = 1 \end{aligned}$$

where $r(y) \equiv r(y, t_1)$ and $\varphi(y) \equiv \varphi(y, t_1)$, Im denotes the imaginary part, and $\varphi(y) = r(y)\, e^{-(i/\hbar)\varphi(y)}$.

Taking the expression for the free kernel K_0 as (Feynman and Hibbs, 1965, p. 42)

$$K_0(xt; zt) = \left(\frac{\alpha}{\pi i}\right)^{1/2} e^{i\alpha(x-z)^2} \tag{4.21}$$

where $\alpha = m/2\hbar\Delta t$, with $\Delta t = t - t_1$, we obtain for condition (i) the relation

$$\begin{aligned}
\text{(i}'\text{)}\quad & \operatorname{Re} G \int_{-\infty}^{+\infty} \cos\left\{\alpha(x-y)^2 - \frac{\pi}{4} + \frac{1}{\hbar}\varphi(y)\right\} r(y)\, dy \\
& + \operatorname{Im} G \int_{-\infty}^{+\infty} \sin\left\{\alpha(x-y)^2 - \frac{\pi}{4} + \frac{1}{\hbar}\varphi(y)\right\} r(y)\, dy \\
& \geq -\left(\frac{\alpha}{\pi}\right)^{1/2} \int_{-\infty}^{+\infty} \cos\left\{\alpha(x-y)^2 - \alpha(x-z)^2 + \frac{1}{\hbar}\varphi(y)\right\} r(y)\, dy
\end{aligned} \tag{4.22}$$

Similarly, condition (ii) becomes

$$\begin{aligned}
\text{(ii}'\text{)}\quad & \int_{-\infty}^{+\infty} dx \left[\operatorname{Re} G \int_{-\infty}^{+\infty} \cos\left\{\alpha(x-y)^2 - \frac{\pi}{4} + \frac{1}{\hbar}\varphi(y)\right\} r(y)\, dy \right. \\
& \left. + \operatorname{Im} G \int_{-\infty}^{+\infty} \sin\left\{\alpha(x-y)^2 - \frac{\pi}{4} + \frac{1}{\hbar}\varphi(y)\right\} r(y)\, dy \right] = 0
\end{aligned} \tag{4.23}$$

I now slightly restrict ψ by requiring that $\psi(z, t_1)$ is a wavepacket, i.e., that $r(z, t_1) = 0$ for $|z| > B$ for some B. Also note that $r(z, t_1)$ must be continuous at z since derivatives of r exist (since ψ satisfies the Schrödinger equation).

I now demonstrate that conditions (i)$'$ and (ii)$'$ are incompatible, as follows. Begin by integrating condition (i)$'$ over some finite region in x, i.e., apply $\int_{-A}^{+A}$, for some A, throughout equation (4.22). The resulting double integrals can then be written as $\int_{-A}^{+A} dx \int_{-B}^{+B} dy$, since $r(y, t_1)$ is a wave packet. But, by previous discussion, the integrands involved here are continuous, which is all that is needed to interchange the order of the above finite integrals (integrals involving infinite limits would require uniform convergence before this would be permitted), so that we have, instead, $\int_{-B}^{+B} dy \int_{-A}^{+A} dx$ throughout. But this can also be expressed as $\int_{-\infty}^{+\infty} dy \int_{-A}^{+A} dx$, again because

$r(y, t_1)$ is a wave packet. Thus, condition (i)′ becomes

$$\int_{-A}^{+A} dx\left[\operatorname{Re} G\int_{-\infty}^{+\infty}\cos\left\{\alpha(x-y)^2-\frac{\pi}{4}+\frac{1}{\hbar}\varphi\right\}r(y)\,dy\right.$$

$$\left.+\operatorname{Im} G\int_{-\infty}^{+\infty}\sin\left\{\alpha(x-y)^2-\frac{\pi}{4}+\frac{1}{\hbar}\varphi\right\}r(y)\,dy\right]$$

$$\geq -\left(\frac{\alpha}{\pi}\right)^{1/2}\int_{-\infty}^{+\infty} dy\, r(y)\int_{-A}^{+A}\cos\left\{\alpha(x-y)^2-\alpha(x-z)^2+\frac{1}{\hbar}\varphi(y)\right\}dx \tag{4.24}$$

Now, by condition (ii)′ we know that, for any $\varepsilon > 0$, no matter how small, there exists an A_0 (z, t) such that for all $A > A_0$, the magnitude of the left-hand side of the above relation is $< \varepsilon$. Therefore, for all such $A > A_0(z, t)$, we have that

$$\left(\frac{\alpha}{\pi}\right)^{1/2}\int_{-\infty}^{+\infty} r(y)\,dy\int_{-A}^{+A}\cos\left\{\alpha(x-y)^2-\alpha(x-z)^2+\frac{1}{\hbar}\varphi(y)\right\}dx \geq -\varepsilon \tag{4.25}$$

Now, the above integral over x can be expressed as

$$\cos\frac{\varphi(y)}{\hbar}\int_{-A}^{+A}\cos\{\alpha(x-y)^2-\alpha(x-z)^2\}\,dx$$

$$-\sin\frac{\varphi(y)}{\hbar}\int_{-A}^{+A}\sin\{\alpha(x-y)^2-\alpha(x-z)^2\}\,dx$$

which, in turn, is equal to

$$-\frac{\sin 2\alpha A(z-y)}{\alpha(z-y)}\cos\left\{\alpha(y^2-z^2)+\frac{1}{\hbar}\varphi(y)\right\}$$

which, in the limit as $A \to \infty$, becomes

$$-\frac{\pi}{\alpha}\delta(z-y)\cos\left\{\alpha(y^2-z^2)+\frac{1}{\hbar}\varphi(y)\right\}$$

This means that, given any $\eta > 0$, there exists an $\bar{A}_0(z, t)$ such that for all $A > \bar{A}_0$,

$$\left|\int_{-\infty}^{+\infty} r(y)\,dy\int_{-A}^{+A}\cos\left\{\alpha(x-y)^2-\alpha(x-z)^2+\frac{1}{\hbar}\varphi\right\}dx\right.$$

$$\left.+\frac{\pi}{\alpha}\int_{-\infty}^{+\infty} r(y)\delta(z-y)\cos\left\{\alpha(y^2-z^2)+\frac{\varphi}{\hbar}\right\}dy\right| < \eta\int_{-\infty}^{+\infty} r(y)\,dy$$

and, therefore, for all $A > \bar{A}_0$, we have

$$\left| \int_{-\infty}^{+\infty} r(y)\, dy \int_{-A}^{+A} \cos\left\{ \alpha(x-y)^2 - \alpha(x-z)^2 + \frac{1}{\hbar}\varphi \right\} dx \right.$$

$$\left. + \frac{\pi}{\alpha} r(z) \cos\frac{\varphi(z)}{\hbar} \right| < \eta c \tag{4.27}$$

where $c \equiv \int_{-\infty}^{+\infty} r(y)\, dy \geq 0$.

Now, equations (4.25) and (4.27) together imply that

$$\frac{\pi}{\alpha} r(z) \cos\frac{\varphi(z)}{\hbar} \leq \varepsilon \left(\frac{\pi}{\alpha}\right)^{1/2} + \eta c \tag{4.28}$$

for all $A > \max(A_0, \bar{A}_0)$ for given arbitrary $\varepsilon > 0$ and $\eta > 0$. Thus, we conclude that, necessarily,

$$\cos\frac{\varphi(z)}{\hbar} \leq 0 \tag{4.29}$$

for all z.

To this point I have shown that the compatibility of conditions (i) and (ii) [or conditions (i)′ and (ii)′], without even yet considering condition (iii), implies the relation (4.29). But this consequence causes a major difficulty, as I now discuss. We know from the structure of the Schrödinger equation that if $\psi = r\, e^{(i/\hbar)\varphi}$ is a solution for all t, then so is $\psi = r\, e^{(i/\hbar)(\varphi + c)}$ (with the same r and φ) a solution for all t, where c is a constant. However, adding c to $\varphi(z)$ will in general make it such that $\cos[\bar{\varphi}(z)/\hbar]$ is not always ≤ 0 (where $\bar{\varphi} \equiv \varphi + c$). Thus, we have this situation: suppose that conditions (i) and (ii) are compatible for a given $\psi = r\, e^{(i/\hbar)\varphi}$. Then necessarily, $\cos[\varphi(z)/\hbar \leq 0$ for all z. But adding a constant to $\varphi(z)$ does not change r (or ρ) for all t, but now, $\cos(\bar{\varphi}/\hbar)$ is not always ≤ 0, so that conditions (i) and (ii) are no longer compatible. Thus, for a *given* $\rho(x, t)$ (for all x, t) we can write

$$\rho(x, t) = \int (P_0 + \Delta)\rho(z)\, dz \equiv \int Q_0 \rho(z)\, dz \tag{4.30}$$

where Q_0 satisfies conditions (i) and (ii) for one choice of $\varphi(z)$ but not for the other, both choices of φ leading to the same $\rho(x, t)$, for all t. But this cannot be, since we could always *define* $P_0 + \Delta \equiv Q_0$ to be given by that value determined by the φ with $\cos(\varphi/\hbar) \leq 0$.

Thus, we cannot construct a physical transition probability in the ECK equation by the second method discussed previously.

I now turn to the first method of identity transformations to see if it will produce a physical transition probability. We have already seen that this method can only yield one real kernel, P_0. The question then is: Does P satisfy conditions (i) and (ii)? The answer to this question can be found from the discussion just finished concerning the second method of constructing kernels. For we have just seen that conditions (i) and (ii) are incompatible [regardless of condition (iii)] for any G, that is, then, for any Δ. In particular, the above two conditions are not satisfied for the choice $G=0$, i.e., for $\Delta \equiv 0$. But then, $Q_0 = P_0$. Thus, P_0 itself cannot satisfy conditions (i) and (ii).

Therefore, it has been proven that if $\rho = \psi\psi^*$, where ψ is any wave packet initially, and ψ is governed by the Schrödinger equation (under zero external field), then ρ cannot satisfy the ECK equation with a physical transition probability. Hence, unobserved position in quantum theory is meaningless by the criterion given here.

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